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A PROGRAM OF AUTOMATIC DIFFERENTIATION

FOR THE BESM COMPUTER

- USSR -

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[This is a translation of a booklet by L. M. Vega,
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The paper contains a general description of a pro-
gram and a method of analytical differentiation for
high-speed electronic digital computer BESM.
Some results of this program application to con-
crete mathematical expressions are presented and
the program block-diagrams are given.

The obtaining of derivatives of complex mathematical expres-
sions is a rather laborious process. The task of obtaining the
derivatives by means of a computer is a problem interesting from
the point of view of programming logical problems; it also has an
applied significance.

In this work there is expounded one of the methods of auto-
matic differentiation of mathematical expressions and there is des-
cribed the arrangement of a corresponding program, developed by a
group of employees of the Institute of Precision Mechanics and
Calculating Techniques, Academy of Sciences USSR for the BESM
computer.

Let us agree that by an expression we designate a line
consisting of letters, numbers, symbols of arithmetical operations,
symbols of elementary functions and the symbols of open and closed
brackets. In order for an expression to have mathematical signifi-
cance, the symbols in it must combine in a definite way; for
example, the number of open brackets must always be equal to the
number of closed, also two symbols of multiplication one after the
other should not occur, etc. Therefore, naturally, we will deal
only with expressions which have mathematical significance.

Let us call parameters or arguments (or simply variables)
the letters and numbers, i.e., all symbols except brackets, symbols
of arithmetical operations, symbols of elementary functions. Any
expression may be regarded as a function of the parameters of
which it is composed. One and the same function may be given in
various ways. For example, the expression $a + b + \sin\sqrt{x}$, which
is obviously a simple function of a, b, x transforms into a complex
function of arguments a, b, x:

$$u_1 = u_1(u_2(a,b), u_3(x))$$

by means of the following set of equalities:

$$u_1 = u_2 + u_3 \; ; \quad u_2 = a + b \; ; \quad u_3 = \sin u_4 \; ; \quad u_4 = \sqrt{x}.$$

Altogether, any expression can, by means of substitution of variables, be put into the trivial form $f(u) = u$ (where u is a complex function of the original arguments). The essence of any method of differentiation is in the utilization of the rule for differentiation of a complex function. The simplicity or complexity of a differentiation method depends on the way the initial expression is presented in the form of a complex function.

At the basis of the offered method lies an algorithm of successive expansion of the initial expression by means of the introduction of new variables in the set of equalities, whose left terms are the new variables, and whose right are easily differentiable elementary tabulated expressions, into which old and new variables enter. The last variable substitutes for the entire expression. The equalities and equalities obtained by means of their differentiation determine uniquely the derivative of the initial expression, which is found by successive substitution into the last differentiated equality of the new variables, i.e., by means of successive elimination of all new variables and a return to the initial parameters.

Let us call an elementary expression or a pair an expression which consists either of two parameters, connected by a sign of arithmetical operation, or of a sign of elementary function and one parameter under the sign of that function.

For example, the expressions cited below are pairs:

$$\sin a \; ; \quad a + b \; ; \quad a - b \; ; \quad \cos a \quad \text{etc.}$$

Our algorithm for expansion of the initial expression into pairs turns out to be equivalent to the algorithm of formula translation (arithmetical operations in the formula) into the sequence of three-address commands in automatic programming. The arithmetical command in a three-address system corresponds to an elementary substitution or pair.

The process of automatic programming is reduced to successive execution of three actions: 1) expanding the expression into elementary pairs, 2) differentiation of elementary pairs (the designations of pairs and of their derivatives form the set of new parameters) and 3) successive exclusion of new parameters form the derivative of the last pair, i.e., the successive substitution of new parameters into the derivative of the last pair.

Before we begin the description of the arrangements of separate parts of the program of automatic differentiation, let us

analyze the accepted system of coding of all the symbols which are found in it. The coding system was conditioned by the arrangement of the BESM computer.

For each symbol eight binary columns are assigned, i.e., in each cell, beginning with the 32nd column, 4 symbols may be placed.

The elementary pairs consist either of three symbols or of two; therefore, each pair takes up no more than one cell.

The parameters, symbols of elementary functions, symbols of arithmetical operations, and brackets, are coded by numbers from 0.1 to 5.15 (see Table 1).

For convenience in coding the elementary pairs $u_0, u_1, \ldots u_k,$ to which the elementary expression is expanded, those are taken which coincide with the address of block cells in which the corresponding pairs are stored. For storing elementary pairs, a special location of cells in the IMD (internal memory device) of the computer, from 6.0 to 10.15 is assigned; hence the pair u_0 is coded as 6.0, the pair u_1 as 6.1, etc. The code of a derivative pair or of a parameter is obtained by adding the code of the pair or parameter to the constant 5.0; for example, u_0 is coded as 11.0, u_1 as 11.1, etc.

For storing derivatives of elementary pairs, another location in the IMD is assigned, from 11.0 to 25.15. The codes of derivative pairs do not coincide with the numbers of the cells in which the appropriate derivatives are stored, since the pair derivative may take up several cells.

In order to avoid coincidence of the codes of parameter derivatives with pair codes, it is possible to construct another system of coding, but that would complicate the perforation of formulas, since more than 8 columns would be needed for coding each symbol.

Each expression subject to differentiation on the computer is brought to a certain standard form. At insertion into the computer, the expression is enclosed in brackets. The numbers which are part of it, with the exception of 0.5 and whole numbers from 1 to 9 inclusive, are replaced by letter symbols. The operation extraction of a root is replaced by raising to the power of 0.5, the complex arguments being enclosed into brackets. The expression, coded with consideration of these rules, is introduced into the IMD of the computer. It should be noted that the rules of coding do not place any limitations on the structure of the expression and are induced by considerations of programming convenience.

The following division of the computer memory has been accepted.

The expression being differentiated is placed in the location of cells from 1.0 to 5.15.

The elementary pairs are formed in the location of cells from 6.0 to 10.15. The derivatives of elementary pairs are obtained in a location of cells from 11.0 to 25.15.

The derivative of the whole expression is formed in the location of cells from 26.0 to 30.15.

Let us examine the first part of the program of expansion into pairs.

In the formation of pairs, i.e., in representing a mathematical expression by means of a succession of pairs, the rule of arithmetical operation must be taken into consideration. Therefore, at first, the bases of powers with exponents are joined into pairs, the symbols of elementary functions with parameters; then pairs of parameters are formed, connected with symbols of the mathematical operations of multiplication or division; and then pairs which contain the symbols of addition or subtraction. In reading a mathematical expression successively symbol after symbol from left to right, the symbols +, -, ., : serve as signals of pair formations which contain a power of a symbol of an elementary function; the symbols +, -,) serve as signals for pair formations which contain the signs ., :; and () serves as a signal for +, -, pair formations.

Example. Let the expression $[\ln \sin x + e^{-(a \cdot x)} \cdot \text{arctg } x]$ be differentiated with respect to x. This expression is expanded into the following pairs;

6.0	sin x	The signal for pair formation is the sign +
6.1	ln 6.0	The signal for pair formation is the sign +
6.2	a·x	The signal for pair formation is the sign)
6.3	0-6.2	The signal for pair formation is the sign· and the preceeding sign
6.4	e✕6.3	The signal for pair formation is the sign ·
6.5	arctg x	The signal for pair formation is the sign)
6.6	6.4·6.5	The signal for pair formation is the sign)
6.7	6.1+6.6	The signal for pair formation is the sign)

The process of representing the expression in the form of a sequence of pairs is best followed by using the instruction diagram of this part of the program (see Instruction Diagram 1).

For a negative number or $-u_1$, if it is the indicator of a power or if it follows directly after an open bracket, the corresponding pair o -x, o- u_1, i.e, o minus this number is put.

In the program for obtaining a sequence of pairs from a mathematical expression, there are two instructions for formation of pairs. One instruction forms pairs from two symbols of the type lnx, the other forms pairs from three symbols, for example, a·x, a+b, etc. Thus, each pair is formed in one of these instructions. The instruction for recording the pair in location 6.0 - 10.15 is a common one. Consequently each pair passes through it.

The instruction for storing symbols in location 26.0-39.15 is also common for all pairs. In this location the symbols are recorded one to each cell, in a sequence from the initial expression, which, during the work, is "read" once from left to right.

Recording in location 26.0-39.15 takes place until the symbol selected from the initial expression turns out to be the signal that a pair may be obtained. This pair is placed in an appropriate place for the location of pairs, and in location 26.0-39.15 there occurs substitution for two or three symbols of the formed pair by one symbol -- the number of the cell in which the pair has been recorded. Later, the computer either uses this symbol for the formation of the next pair, or, depending on the instruction, continues to select the next symbols of the original expression.

The intermediate recording of symbols in location 26.0-39.15 simplifies to a considerable degree the program of algorithmic expansion into pairs, since it liberates it from the complicated work of substituting variables within the initial expression and allows it to move along in only one direction.

In the program there is an instruction for the "destruction" of brackets, which operates when there turns out to be only one symbol within the brackets. If, for example, we come across the combination (u + a), then, after formation of the pair u + a and the substitution for it of u_1, we obtain (u_1). These brackets are destroyed.

A derivative pair which has passed through one of the above-mentioned instructions is recorded in the location of cells from 59.0 to 59.4. The instruction for storing the derivative of the given pair in this location is common for all pairs. Simplification of the obtained expression of the derivative is then performed, since the differentiation is performed according to formulas of a general type; for example, for differentiating x^a or a^x, the base and the exponent are substituted into the general formula for the derivative of a power:

$$(u^v)' = u^v(v'\ln u + \frac{v}{u}u').$$

The simplification is further reduced by destroying brackets, unnecessary for the expression of the derivative of a concrete pair, symbols of elementary operations, zero-addends, unit-multipliers and products into which zero enters as a factor. For each instruction for derivative derivation, there is a particular instruction for simplification of the derivative (see Instruction Diagram 2). Such simplifications as collecting terms and reduction of fractions are not performed.

The derivatives which have passed through the simplification instructions are recorded in the location of cells from 11.0 to 25.15; furthermore, this record is realized in general for all instruction pairs. If the derivative of a pair turns out to be zero, then, in the corresponding cell of location 11.0-25.15, the code for zero is recorded. In the same instruction, the relative address of the origination of the derivative according to 11.0-25.15 (obviously $N_{rel} = N_{cell} - 11.0$) is obtained. This address is recorded in the last 8 columns of a cell of location 6.0-10.15,

where the corresponding elementary pair of this derivative is located. Thus, each elementary pair has the address of its derivative. For example, pair u_2 has the form $a \cdot x$ in cell 6.2 (see example, page 6), has $o \cdot x + a \cdot 1$ in cell 59.0 and a in cell 11.3.

As a result of treating each elementary pair by the second part of the program, a series of elementary derivatives is obtained.

As an example we will cite the contents of the location of cells from 6.0 to 10.15 and the location of cells from 11.0 to 25.15 which will be obtained as a result of the work of the second part of the program for automatic differentiation (see Table 2).

By the beginning of the third part of the program pairs are recorded in the location 6.0-10.15; furthermore, in the first eight columns of cells of this location there are decreased by 11.0, the numbers of the cells in which the corresponding derivatives originate, and in location 11.0-25.15 the derivatives of the pairs are recorded.

The work of the third part of the program terminates with obtaining the derivative of the initial mathematical expression, recorded in the location 26.0-30.15.

We know that the last pair replaces the initial expression; consequently, the derivative of the last pair equals the derivative of the initial expression. Therefore it is natural to obtain the derivative of the initial formula, expressed in the initial parameters, by subsequent substitution into the derivative of the last pair of values for all new parameters which are a part of it and which signify pairs and derivatives of pairs.

Let us describe in more detail the process of obtaining the derivative. In the last finished cell of the location 6.0-10.15 there is, decreased by 11.0 (relative), the number of the cell in which the derivative of the last pair originates. The obtained number allows selecting, from location 11.0-25.15, the derivative of the last pair and bringing it into location 1.0-5.10. From location 1.0-5.10 consecutively 8 columns are taken, which are checked for coincidence with the codes of the parameters that denote pairs or derivatives of pairs. If it turns out that there is no such coincidence, the computer registers the code of the symbol registered in these eight columns in location 26.0-30.15. However, if in the selected 8 columns there is a parameter code which denotes a pair (or derivative pair), then, from location 6.0-10.15, the pair corresponding to that parameter (or from location 11.0-25.5 the derivative of the pair) is selected and is transmitted into location 63.11-63.15, where it is analyzed and then recorded in location 26.0-30.15 in brackets or without brackets depending on the results of the analysis (see Instruction Diagram 3).

Having exhausted all elements of location 1.0-5.15, the computer replaces them by elements of location 26.0-30.15 and

repeats the same cycle. This continues until, in location 26.0-30.15, the derivative of the initial mathematical expression is obtained, i.e., until the expression recorded in location 26.0-30.15 finally contains neither a parameter which is a pair nor a derivative of a pair. The computer judges whether there still are parameters which denote pairs or derivatives of pairs in location 26.0-30.15 according to register 0.10. If 0.10 is an occupied cell, such parameters are still in the expression and the repetition of cycles is not yet finished. If 0.10 is an empty cell, then the work is considered finished and the computer is stopped.

The basic instructions in this part of the program are: the instructions for selection of the element form location 1.0-5.15, for checking it for a parameter which denotes a pair or the derivative of a pair and storing the pair or derivative of the pair selected from the appropriate location massifs 6.0-10.15 or 11.0-25.15 in location 63.11-63.15; the instruction for recording the element in location 26.0-30.15 as well as instructions for analysis of the elements of location 63.11-63.15 for +, -, comparing previous elements for a sign of an elementary function, x, :, $\gamma\!\!\!\!\sim$ and comparing the subsequent elements for x, :, $\cancel{X}$.

As a result of the work of this part of the program for the above-examined example, we will obtain the following:

26.0	(1 : sin
26.1	x) : cos
26.2	x + e.$\cancel{x}$
26.3	(- a.
26.4	x)·(
26.5	-a)·
26.6	log e·arctg
26.7	x + 1:
26.8	(1 + x
26.9	$\cancel{x}$ 2).
26.10	e $\cancel{x}$ (-
26.11	a · x)

It is clear that by storing the obtained derivative in the cell location 1.0-5.15 and changing the same program again, the second, third, etc., derivative may be obtained.

In order to find Taylor coefficients, to solve problems in the analysis of curves and surfaces, to find maxima, inflection points, curvature and torsion, etc., programs are needed which, in their structure, would be closely related to the program analyzed here, since it is necessary to find the values of a function and its derivative at a given point.

If in expanding an expression to pairs one successively finds the numerical values of the pairs, then the numerical value of the last pair will be the numerical value of the whole expression at some given parameters.

Thus, the first part of the program for automatic differentiation can change into a program for computation of the value of a mathematical expression at a certain point, and the whole program may be utilized for the computation of derivatives of a mathematical expression at that point. For example, in order to obtain the numerical value of k derivative, it is necessary that the program be repeated k times, and the first part of the program k + 1 times.

The method of decomposing an expression into pairs will help to solve the following interesting general problem: to compose a program O_F which, operating with the commands of any program P_f that computes a function $f(x)$, would develop a program P_{Ff} that computes a function $F(f(x))$, where F_2 is a certain operater. In particular, by F one can mean differentiation or integration.

The utilization in a somewhat different manner of the parts of the above-described program for differentiation of mathematical expressions will allow one to solve the problem of differentiating some class of programs.

In actuality, the sequence of the pairs of location 6.0-10.15 is nothing else but a program for computing the function f which was originally expressed in the customary mathematical symbolism. The two last parts of the program for differentiation allow one to obtain a mathematical expression for the derivative of the function f. It is evident that, if the obtained expression of the derivative is expanded into elementary pairs, we will obtain a program for computation of the derivative f'. For this, we have only to apply the first part of the program for differentiation.

Thus, in this process we proceeded form a certain program P_f; we applied the operator Q_D to it and we obtained a new program P_{Df} -- a program for the computation of the derivative function of f.

As a conclusion, we will cite a number of examples of functions and their derivatives obtained on the BESM computer.

Example 1

$$f(x) = (\text{arctg ln } (x^n - 1) + e^x) : (\cos x^3 - \sin(x^3 + \arcsin x))$$

$$f'(x) = (((1 : (1+(\ln(x^n - 1))^2)) \cdot (1 : (x^n-1)) \cdot x^n \cdot n : x + e^x \cdot \ln e) \cdot (\cos(x^3) -$$
$$-\sin(x^3 + \arcsin x)) -((-\sin(x^3) \cdot$$
$$\cdot x^3 \cdot 2 : x) - \cos(x^3 + \arcsin x) \cdot$$
$$\cdot (x^3 \cdot 3 : x + 1 : (1 - x^2)^{1/2})).$$
$$\cdot (\text{arctgln}(x^n-1) + e^x)) : (\cos(x^3) -$$
$$- \sin(x^3 + \text{arc sin } x))^2$$

Example 2

$$f(x) = (\text{arctg ln tgcos}(x + \text{arc sin } x))$$

$$f'(x) = (1 : (1+(\text{lntgcos}(x+\text{arcsinx}))^2)) \cdot$$

$$\cdot (1 : \text{tgcos}(x + \text{arcsinx})) \cdot 1 : (\text{coscos }(x+ \text{arcsinx}))^2 \cdot$$

$$\cdot (-\sin(x + \text{arcsinx}) \cdot (1 + 1: (1 - x^2)^{1/2}))$$

Example 3

$$f(x) = (\ln(x + \ln(x + \ln(x + \ln(x + \ln x)))))$$

$$f'(x) = (1 : (x + \ln(x + \ln(x + \ln(x+ \ln x)))) \cdot$$

$$\cdot (1 +(1 : (x + \ln(x + \ln(x+ \ln x))) \cdot$$

$$\cdot (1+(1:(x+\ln(x+\ln x))) \cdot (1+(1:(x+\ln x)) \cdot (1+1:x)))) .$$

In a careful analysis of the derivative formulas obtained on the computer, it will be seen that further simplifications in them are possible. This problem may also be solved by means of the computer.

Table 1

Symbol	Code	Symbol	Code	Symbol	Code
+	0.1	0	2.0	t	3.15
-	0.2	1	2.1	u	4.0
x	0.3	2	2.2	v	4.1
:	0.4	3	2.3	w	4.2
(	0.5	4	2.4	x	4.3
)	0.6	5	2.5	y	4.4
x (degree)	0.7	6	2.6	z	4.5
=	0.8	7	2.7		
sin	0.9	8	2.8		
cos	0.10	9	2.9		
tg	0.11	e	2.10		
ctg	0.12	π	2.11		
exp	0.13	,	2.12		
arsin	0.14	a	2.13		
arccos	0.15	b	2.14		
arctg	1.0	c	2.15		
ch	1.1	d	3.0		
sh	1.2	e	3.1		

ln	1.3	r	3.2	
√	1.4	q	3.3	
∛	1.5	h	3.4	
α	1.6	i	3.5	
β	1.7	$\varkappa$	3.6	
γ	1.8	l	3.7	
δ	1.9	m	3.8	
η	1.10	n	3.9	
υ	1.11	o	3.10	
φ	1.12	p	3.11	
ψ	1.13	$ξ$	3.12	
w	1.14	r	3.13	
σ	1.15	s	3.14	5.15

Table 2

60	sin x	0
1	ln 6.0	1
2	α·x	3
3	0 — 6.2	4
4	s×6.3	5
5	arctg x	7
6	6.4·6.5	10
7	6.1 + 6.6	12
8		
9		
10		
11		
12		

11.0	cos x
1	(1 : 6.0
2	) x 11.0
3	a
4	~11.2
5	e x 6.3
6	11.3 · log e
7	1 : (1
8	+x x 2
9	)
10	11.4 ·6.5 +
11	11.5 ·6.4
12	11.1 + 11.6
13	
14	

Instruction Diagram 1 of the program for
representation of a mathematical expres-
sion by means of a sequence of pairs.

The notation $\alpha(\beta, \gamma)$ should be understood as follows:
α is the number of the instruction, β is the number of the instruc-
tion to which we pass in case of positive results of checking, and
γ is the number of the instruction to which we pass in the
opposite case. The notation $\alpha(\beta - \gamma)$ means that after the instruc-
tion N_α one should pass to the instruction N_β, and then to the
instruction N_γ .

1(2,2)	The selection of the next symbol
2(1,3)	Checking for the sign of an open bracket
3(1,4)	Checking for the sign of an elementary function and parameter
4(1,5)	Checking for the sign of a power
5(28,6)	Checking for a multiplication and division sign
6(16,7)	Checking for an addition sign
7(8,10)	Checking for a subtraction sign
8(9,18)	Checking the preceeding sign for an open bracket
9(1,20)	Checking the subsequent sign for an open bracket.
10(11,13)	Checking for the sign of a closed bracket
11(12,17)	Checking upwards over one for the sign of an open bracket,

12(1,1)   Destruction of brackets
13(14,15)   Checking for the sign of the end of a formula (empty)
14(0,0)   Stop (correct)
15(0,0)   Stop (incorrect)
16(1,21)   Checking over one symbol upwards for the sign of an
                open bracket
17(11,11)   Recording a pair in location 6.x. Exchange of variables
18(9,16)   Checking the preceeding sing for the sign of a power
19(1,26)   Checking for multiplication and division the last sign
                selected from the expression
20(1,1)   Recording the pair in location 6.x. Exchange of variables
21(22,23)   Checking over one symbol upwards for a sign of an
                elementary function
22(16,16)   Recording the pair in location 6.x. Exchange of variables
23(24,22)   Checking for the sign of subtraction
24(25,19)   Checking the sign before a minus for a power
25(2,2)   Recording a pair in the location 6.x. Exchange of
                variables
26(28,28)   Recording a pair in the location 6.x. Exchange of
                variables
27(24,26)   To check over one symbol upwards for a subtraction
                sign
28(1,27)   Checking over one symbol upwards for signs of addition
                and open bracket.

                Instruction Diagram 2 of the program for obtaining
                derivatives of elementary pairs

1(2,2)   Selection of the pair from location 6.0-10.15
2(3,4)   Checking for emptiness in the cells of location
                6.0-10.15
3(0,0)   Print shop
4(5,29)   Checking for pairs which consist of two symbols
5(6,6)   Formation of derivatives from the parameters which
                enter the given pair
6(7,11)   Checking the first symbol of the pair for sin
7(8-9)   Formation of a derivative from sin v, cos v, exp v.
8   Recording derivatives in general form into the
                location of cells 59.0-59.4
9(10,10)   Simplification of the expression of derivatives from
                sin v, cos v, exp v
10(1,1)   Recording the final expression of the derivative of a
                pair in location 11.0-25.15 and recording the ordinal
                number of the cell in which the derivative originates
                in the last 8 columns of the cells of location
                6.0-10.15 which correspond to the given pair
11(7,12)   Checking the 1st symbol of the pair for cos
12(7,13)   Checking the 1st symbol of the pair for exp.
13(14,16)   Checking the 1st symbol of the pair for ln

14(8-15)   Formation of a derivative of ln v

15(10,10)   Simplification of the expression for a derivative of ln v

16(17,19)   Checking the first symbol of the pair for tg

17(8-18)   Formation of the derivative of tg v

18(10,10)   Simplification of the expression of the derivative of tg v and ctg v

19(20,21)   Checking the first symbol of the pair for ctg v

20(8-18)   Formation of a derivative from ctg v

21(22,24)   Checking of the first symbol for arc sin

22(8-23)   Formation of a derivative of arc sin v and arc cos v

23(10,10)   Simplification of the expression for a derivative of arc sin v and arc cos v

24(22,25)   Checking the first symbol of the pair for arc cos

25(26,28)   Checking the first symbol of the pair for arctg

26(8-27)   Formation of a derivative of arctg v

27(10,10)   Simplification of the expression of the derivative of arctg v

28(0,0)   Stop

29(30,30)   Formation of derivatives from the parameters which enter into the given pair

30(31,33)   Checking for multiplication sign

31(8-32)   Obtaining a derivative from $u \cdot v$

32(10,10)   Simplification of the expression for a derivative of the product

33(34,36)   Checking for a division sign

34(8-35)   Formation of a derivative from u:v

35(10,10)   Simplification of the expression for the derivative of the quotient

36(37,39)   Checking for the sign of a power

37(8-38)   Formation of a derivative from u^v

38(10,10)   Simplification of the expression for a derivative of the power

39(8-40)   Formation of a derivative of $u \pm v$

40(10,10)   Simplification of the expression of the derivative of the sum and difference

Instruction Diagram 3 of the program for synthesis of the derivative formula

1(2,2)   To select the derivative of the last pair and record it in location 1.0-5.15

2(3,3)   To select the next element from the location 1.0-5.15

3(7,4)   To check it for being empty

4(5,22)   To check it for "u" or "u'", i.e., is the element a pair or the derivative of a pair?

5(12,12)   To select the value u (from location 6.0-10.15) or u' (from location 11.0-25.15) and record it in

	location 63.11-63.15
5(0,0)	The printing of results (location 6.0-10.15, 11.0-25.15, 26.0-30.15). Stop
7(6,21)	To check for the end of substitution, i.e., for the absence in 26.0-30.15 of u and u'
8(16,18)	To check the element before the selected element in location 1.0-5.15 for a sign of an elementary function
9(16,17)	To check the element next after the selected element in location 1.0-5.15 for a sign of a power
10(9,15)	To check location 63.11-63.15 for two elements
11(16,10)	To check the first element in 63.11 for a subtraction sign
12(17,11)	To check the location 63.11-63.15 for one element
13(19,8)	To check the location 63.11-63.15 for signs of addition and subtraction
14(16,16)	To check the element following after the selected element in location 1.0-5.15 for the sign of a power
15(16,14)	To check the element next after the selected element in location 1.0-5.15 for a division sign
16(2,2)	To record the content of cells 63.11-63.15 in location 26.0-30.15 with brackets
17(2,2)	To record the content of cells 63.11-63.15 in location 26.0-30.15 without brackets
18(16,17)	To check the element next after the selected element in location 1.0-5.15 for a sign of a power
19(16,20)	To check the element before the selected element in location 1.0-5.15 for a sign of multiplication, division, degree or elementary function
20(16,17)	To check the element following after the selected element in location 1.0-5.15 for signs of multiplication or division
21(2,2)	To transmit the whole phrase from location 26.0-30.15 into location 1.0-5.15. To clear location 26.0-30.15
22(2,2)	To record the element selected from location 1.0-5.15 into location 26.0-30.15 without a bracket

END

FOR REASONS OF SPEED AND ECONOMY

THIS REPORT HAS BEEN REPRODUCED

ELECTRONICALLY DIRECTLY FROM OUR

CONTRACTOR'S TYPESCRIPT